Box-Jenkins Transfer Function Modelling for Reliable Determination of VO$_2$ Kinetics in Patients with COPD

Joren Buekers $^{1,2}$, Jan Theunis $^1$, Alberto Peña Fernández $^2$, Emiel F. M. Wouters $^{3,4}$, Martijn A. Spruit $^{3,4,5,6}$, Patrick De Boever $^{1,7}$ and Jean-Marie Aerts $^{2,*}$

1 Health unit, Flemish Institute for Technological Research (VITO), 2400 Mol, Belgium; joren.buekers@vito.be (J.B.); jan.theunis@vito.be (J.T.); patrick.deboever@vito.be (P.D.B.)
2 Measure, Model & Manage Bioresponses (M3-BIORES), Department of Biosystems, KU Leuven, 3000 Leuven, Belgium; alberto.penafernandez@kuleuven.be
3 Department of Research and Education, Centre of Expertise for Chronic Organ Failure (CIRO), 6085 NM Horn, The Netherlands; e.wouters@mumc.nl (E.F.M.W.); martijnspruit@ciro-horn.nl (M.A.S.)
4 Department of Respiratory Medicine, Maastricht University Medical Centre, 6229 HX Maastricht, The Netherlands
5 REVAL—Rehabilitation Research Center, BIOMED—Biomedical Research Institute, Faculty of Rehabilitation Sciences, Hasselt University, 3500 Diepenbeek, Belgium
6 NUTRIM School of Nutrition and Translational Research in Metabolism, Maastricht University Medical Centre, 6211 LK Maastricht, The Netherlands
7 Centre for Environmental Sciences, Hasselt University, 3500 Diepenbeek, Belgium
* Correspondence: jean-marie.aerts@kuleuven.be

Abstract: Oxygen uptake (VO$_2$) kinetics provide information about the ability to respond to the increased physical load during a constant work rate test (CWRT). Box-Jenkins transfer function (BJ-TF) models can extract kinetic features from the phase II VO$_2$ response during a CWRT, without being affected by unwanted noise contributions (e.g., phase I contribution or measurement noise). CWRT data of 18 COPD patients were used to compare model fits and kinetic feature values between BJ-TF models and three typically applied exponential modelling methods. Autocorrelation tests and normalised root-mean-squared error values (BJ-TF: 2.8 ± 1.3%; exponential methods A, B and C: 10.5 ± 5.8%, 11.3 ± 5.2% and 12.1 ± 7.0%; $p < 0.05$) showed that BJ-TF models, in contrast to exponential models, could account for the most important noise contributions. This led to more reliable kinetic feature values compared to methods A and B (e.g., mean response time (MRT), BJ-TF: 74 ± 20 s; methods A-B: 100 ± 56 s–88 ± 52 s; $p < 0.05$). Only exponential modelling method C provided kinetic feature values comparable to BJ-TF features values (e.g., MRT: 75 ± 20 s). Based on theoretical considerations, we recommend using BJ-TF models, rather than exponential models, for reliable determinations of VO$_2$ kinetics.

Keywords: chronic obstructive pulmonary disease; COPD; oxygen uptake; VO$_2$; kinetics; exercise testing

1. Introduction

Chronic obstructive pulmonary disease (COPD) is a common lung disease characterised by persistent airflow limitations due to a combination of airway and alveolar abnormalities. COPD is associated with high morbidity and mortality rates, posing a high economical and societal burden [1]. The most important symptom of COPD is dyspnoea during daily activities, but later also at rest. As a result, patients become physically less active, which is the start of a vicious circle of
physical deconditioning [2]. Physical capacities can be assessed using standard exercise tests like a cardiopulmonary exercise test (CPET, incremental cycling load) or a constant work rate test (CWRT, constant cycling load) [3]. During a CPET, oxygen uptake ($VO_2$, ml.min$^{-1}$) increases linearly until the maximal or peak $VO_2$ value is reached. This maximal (or peak) $VO_2$ value, the maximal load reached during CPET and the endurance time during CWRT are the most frequently used metrics for evaluating physical capacities. However, there is additional information, besides these maximal values, that can be extracted from the breath-by-breath $VO_2$ time series that are generated during these tests.

During a CWRT, there is typically a 3-phase $VO_2$ response towards the step increase of cycling load (W). During phase I, the cardiodynamic phase, $VO_2$ shows an instant response driven by an increased cardiac output and pulmonary blood flow (Figure 1). In phase II, the primary component of the response, $VO_2$ increases exponentially, similar to the working muscle oxygen uptake. In phase III, $VO_2$ can reach a steady-state or can steadily keep increasing, i.e., $VO_2$ slow component (Figure 1), depending on the exercise intensity [4]. Kinetic analyses of the $VO_2$ response aim to quantify the speed and amplitude of the primary (phase II) response using a combination of features. Time delay (TD), time constant (TC) and mean response time ($MRT = TD + TC$) assess the speed of the response. Response amplitude (Amp) and steady state gain ($SSG = Amp.\Delta W^{-1}$) assess the magnitude of the response, with $\Delta W$ indicating the step increase of cycling load at the start of the exercise test (Figure 1). These features quantify the ability to respond to the increased physical load during a CWRT [4]. Generally, COPD patients exhibit a slower $VO_2$ response than healthy subjects [5,6], but supplemental oxygen [7], medication use [8,9] and exercise training [10], can speed up the $VO_2$ response to the step increase in cycling load.

Typically, exponential models are used to extract these kinetic features from the $VO_2$ time series. Three mono-exponential modelling methods have been described in COPD literature, which will be referred to as method A [6,8], B [10,11] and C [9,12]. Method A assumes there is no delay in the $VO_2$ response (TD = 0 s), whereas methods B and C estimate a TD value as one of the model parameters. Method B uses data starting from $t = 0$ s (load onset), while method C aims to exclude the contribution of the cardiodynamic phase by only using data starting from $t = 20$ s. Nevertheless, these exponential models are all still influenced by other unwanted noise contributions, such as measurement noise or cardiac output dynamics that are not related to the increased working muscle oxygen uptake.

In contrast to the generally applied exponential models, Box-Jenkins transfer function (BJ-TF) models can extract the same kinetic features [13] without being affected by these unwanted noise contributions. BJ-TF models are able to separate the unwanted contributions from the noise-free primary $VO_2$ response (i.e., phase II). Therefore, extraction of the kinetic features will not be influenced by the unwanted contribution of these additional components, and will only reflect the working muscle oxygen uptake. To the authors’ knowledge, BJ-TF models have not yet been used to extract kinetic features from CWRT data. Therefore, the aim of this study was to compare model fits and kinetic feature values between BJ-TF models and the three different exponential modelling methods that were identified in COPD literature.
2. Materials and Methods

2.1. Study Design and Participants

Twenty COPD patients (GOLD II-IV) were recruited at CIRO (Horn, the Netherlands) during a standard baseline assessment prior to pulmonary rehabilitation [14]. Two patients were excluded from analyses because they failed to perform a CWRT during the baseline assessment. Demographics, partial pressure of oxygen and carbon dioxide, post-bronchodilator pulmonary function data (e.g., forced expiratory volume in 1 second, forced vital capacity and transfer factor for carbon monoxide) and modified Medical Research Council dyspnoea scores were collected during the standard baseline assessment at CIRO. The study was approved by the Medical Research Ethics Committees United (NL58079.100.16, MEC-U, the Netherlands) and all patients provided written informed consent.

2.2. Exercise Testing

Patients performed a CPET during the first day and a CWRT during the second day of the baseline assessment. The tests were performed on an electrically braked cycle ergometer and VO₂ data were collected breath-by-breath (Oxycon Pro, Carefusion, Houten, the Netherlands). Both tests started with 3 minutes of rest, followed by 3 minutes of unloaded cycling (W = 0). During CPET, the load then increased 1 Watt per 4, 6 or 12 seconds, depending on the relative fitness of the patient. During CWRT, the load remained constant at 75% of the maximal load obtained during the CPET. In both tests, patients were asked to keep their cycling speed constant between 60 and 70 rotations per minute and to cycle until limitation (exhaustion and/or severe dyspnoea). The tests ended with a 3-minute recovery period. CWRT time series data were used for the analyses described below.

2.3. Data Pre-Processing

Deviating breath values, identified as data samples deviating more than 3 standard deviations from the local mean (i.e., 31 s centred moving average), were deleted. An integrated random walk smoothing algorithm (CAPTAIN toolbox [15]) was used to converse the non-equidistant breath-by-breath VO₂ data into a smoothed 1 Hz time series, while still preserving the dynamics of the time series. Data after 180 s of loaded cycling were discarded to exclude any potential contribution of the VO₂ slow component.

Figure 1. Graphical representation of the kinetic features that quantify the phase II response. The blue line represents the theoretical VO₂ response, the orange line shows the phase II contribution that should be modelled, the black dashed line visualises the load increase at t = 0 s. The blue and orange lines coincide during phase II. A slow component is visible in phase III. TD, Time delay; TC, time constant; MRT, mean response time; Amp, response amplitude; ΔW, the step increase in cycling load at t = 0 s; and SSG, steady state gain.
2.4. Kinetic Analyses

The dynamic response of physiological variables towards an increase in physical load can be described by a number of features (Figure 1). These features were extracted using both exponential models and BJ-TF models. The following first order model was used as a basis for exponential modelling:

\[
\begin{align*}
0 \leq t & \leq TD : \Delta VO_2(t) = 0, \\
TD < t \leq 180 : \Delta VO_2(t) = Amp \left( 1 - e^{-(t-TD)} \right),
\end{align*}
\]

where \( \Delta VO_2(t) \) is the difference between the pre-processed \( VO_2 \) time series and the baseline \( VO_2 \) level (\( VO_2,\text{baseline} \)), i.e., \( \Delta VO_2(t) = VO_2(t) - VO_2,\text{baseline} \). \( VO_2,\text{baseline} \) is calculated as the mean \( VO_2 \) value of the last 30 s of unloaded cycling. The model parameters (\( Amp, TD \) and \( TC \)) were estimated using non-linear least squares (Matlab 2015b, MathWorks Inc., Natick, Massachusetts, USA). Three mono-exponential modelling methods have been described in COPD literature. They will be referred to as method A [6,8], B [10,11] and C [9,12]. Method A assumes there is no TD in the \( VO_2 \) response (\( TD = 0 \) s). Method B estimates the three model parameters using data from \( t = 0 \) s (load onset) until \( t = 180 \) s, whereas method C only uses data between \( t = 20 \) s and \( t = 180 \) s to exclude the contribution of the cardiodynamic phase (phase I).

Time evolutions and autocorrelation tests of the model residuals were checked to assess whether the model residuals retained any meaningful structure. Remaining structure in the model residuals indicates that the residuals contain information that is not captured by the model. The relative amplitude of these residuals was assessed by the normalised root-mean-squared error (\( NRMSE \)), defined as the root-mean-squared error divided by the amplitude of the \( VO_2 \) response:

\[
NRMSE = \sqrt{\frac{\sum_{t=1}^{n} \text{residuals}(t)^2}{n}} \cdot \text{Amp}^{-1}.
\]

A discrete-time, single-input, single-output BJ-TF model with time-invariant parameters can generally be written as follows:

\[
y(t) = \frac{B(z^{-1})}{A(z^{-1})} u(t - TD) + \frac{C(z^{-1})}{D(z^{-1})} \epsilon(t),
\]

where \( y(t) \) is the model output at time \( t \), \( u(t) \) is the model input at time \( t \), \( z^{-1} \) is the backwards shift operator (i.e. \( z^{-s} y(t) = y(t-s) \)), \( \epsilon(t) \) is zero mean white noise and \( A(z^{-1}), B(z^{-1}), C(z^{-1}) \) and \( D(z^{-1}) \) are polynomials containing the model parameters:

\[
\begin{align*}
A(z^{-1}) &= 1 + a_1 z^{-1} + \cdots + a_m z^{-m}, \\
B(z^{-1}) &= b_0 + b_1 z^{-1} + \cdots + b_n z^{-n}, \\
C(z^{-1}) &= 1 + c_1 z^{-1} + \cdots + c_p z^{-p}, \\
D(z^{-1}) &= 1 + d_1 z^{-1} + \cdots + d_q z^{-q}.
\end{align*}
\]

Thus, the full BJ-TF model (3) is a combination of a ‘noise-free’ system model (first term) and a noise model (second term). As phase II is known to follow a first-order exponential response,
the system model was defined as a first order model with one gain parameter. This transformed the full model (3) as follows:
\[
\Delta V O_2(t) = \frac{b_0}{1 + a_1 z^{-1}} W(t - TD) + \frac{C(z^{-1})}{D(z^{-1})} e(t),
\] (5)
where the input variable \( W(t) \) was defined as:
\[
t \leq 0 : \quad W = 0 \text{ Watt},
\]
\[
t > 0 : \quad W = 75\% \text{ of the maximal load reached during CPET}.
\]

The order of the noise model, given by the order of polynomial \( D(z^{-1}) \), was unknown and was therefore identified by comparing the performances of models with noise models up to order three in \( C(z^{-1}) \) and/or \( D(z^{-1}) \). Full model fits were inspected visually and the residuals were checked using autocorrelation tests and NRMSE values. In addition, the full models with different noise structures were compared by their Young Identification Criterion (YIC) value [16]. YIC combines goodness of fit and reliability of the parameter estimations, while penalising for over-parameterisation. A more negative YIC value indicates a better model performance. The noise model structure will be referred to as \([p \ q]\), where \( p \) and \( q \) are the orders of polynomials \( D(z^{-1}) \) and \( Z(z^{-1}) \), respectively.

The parameters of the full model were estimated using the refined instrumental variable approach of the CAPTAIN toolbox in Matlab [16,17]. From the system model parameters \( a_1 \) and \( b_0 \), TC and SSG were calculated as follows:
\[
TC = \frac{-\Delta t}{\log(-a_1)},
\] (6)
\[
SSG = \frac{b_0}{1 + a_1},
\] (7)
where \( \Delta t \) is the sampling time (\( = 1 \text{ s} \)). MRT was calculated as the sum of TD and TC, Amp was calculated by multiplying SSG with the load increase (\( \Delta W \)).

2.5. Statistical Analysis

Patient characteristics were summarised for all patients as mean and standard deviation (SD). NRMSE and kinetic feature values were summarised as median and interquartile range (IQR). Wilcoxon signed-rank tests were used to look for differences in NRMSE and kinetic feature values as calculated with the different exponential modelling methods and BJ-TF modelling. A \( p \)-value lower than 0.05 was considered to provide statistical significance. All analyses were performed using Matlab 2015b (MathWorks Inc., Natick, Massachusetts, USA).

3. Results

Eighteen patients (14 male and four female) performed a CWRT. Table 1 summarises the clinical characteristics of the participants. On average, the elderly patients had a moderate-to-severe degree of airflow limitation, an impaired diffusing capacity and a slightly overweight body mass index. None of the patients were on long-term oxygen therapy. A median of 1.5 (\( \pm 2 \)) deviating breath values were deleted per patient.
Table 1. Patient characteristics.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Value Mean (SD)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age (years)</td>
<td>63 (7)</td>
</tr>
<tr>
<td>Body mass index (kg/m²)</td>
<td>26 (5)</td>
</tr>
<tr>
<td>Forced expiratory volume in 1 second (FEV₁) % predicted</td>
<td>48 (14)</td>
</tr>
<tr>
<td>Forced vital capacity (FVC) % predicted</td>
<td>105 (18)</td>
</tr>
<tr>
<td>FEV₁/FVC (%)</td>
<td>37 (13)</td>
</tr>
<tr>
<td>Transfer factor for carbon monoxide % predicted</td>
<td>55 (15)</td>
</tr>
<tr>
<td>Partial pressure of oxygen (kPa)</td>
<td>8.8 (1.8)</td>
</tr>
<tr>
<td>Partial pressure of carbon dioxide (kPa)</td>
<td>5.3 (0.7)</td>
</tr>
<tr>
<td>Modified Medical Research Council dyspnoea scores (scores 1-2-3)</td>
<td>2-14-2</td>
</tr>
<tr>
<td>CPET maximal cycling load % predicted [18]</td>
<td>50 (23)</td>
</tr>
<tr>
<td>CPET maximal oxygen uptake (ml/min)</td>
<td>1133 (231)</td>
</tr>
<tr>
<td>CPET maximal oxygen uptake % predicted [18]</td>
<td>55 (21)</td>
</tr>
<tr>
<td>CWRT cycling load (W)</td>
<td>57 (16)</td>
</tr>
<tr>
<td>CWRT endurance time (s)</td>
<td>243 (81)</td>
</tr>
</tbody>
</table>

CPET, cardiopulmonary exercise test; CWRT, constant work rate test.

3.1. Exponential Modelling

Figure 2 shows an example of the three exponential modelling methods fitted to data of the same patient. NRMSE values differed between the three methods (method A: 10.5 ± 5.8; method B: 11.3 ± 5.2; method C: 12.1 ± 7.0; p < 0.05 for all pairwise comparisons). As shown in Figure 2, method C is the only method that can completely surpass the cardiodynamic phase.

The exponential model residuals contained remaining structure, as can be seen by plotting the relative residual values in function of time and the autocorrelation coefficients from lag 1 to 10 (example shown in Figure 3a). The same behaviour was obtained for all patients for all three methods.

![Figure 2](image_url)

Figure 2. Example plot of VO₂ data and the three different exponential modelling methods. The estimated time delay (TD) values for methods B and C were 8.3 s and 20.3 s, respectively.

3.2. Box-Jenkins Transfer Function Modelling

Based on the full model fits, YIC values and NRMSE values (see below and Discussion), a first order system model with a [2 0] noise model order was selected and used for comparing kinetic feature values between exponential and BJ-TF models in the next section. An example of applying this BJ-TF model is shown in Figure 4.
BJ-TF models with first order noise models (i.e., first order in $D(z^{-1})$) resulted in visually inappropriate model fits. Noise models of order [1 0] and [1 1] led to the full model (i.e., system + noise model) lagging behind the measured VO$_2$ data. This lag was solved in the noise models of order [1 2] and [1 3]. However, for both these noise model structures the full model fits were completely off for some patients. Similarly, noise model orders [2 3] and [3 3] led to bad fits for some patients. YIC and NRMSE values of the remaining noise model orders are summarised in Table 2. Second order noise models had more negative (i.e., better) YIC values, but slightly higher residual values compared to third order models. Example residual and autocorrelation plots for [2 0] and [3 2] noise models are shown in Figure 3b,c, respectively. These noise model orders were chosen as examples because they had the lowest YIC value of all second and third order noise models, respectively. All second order noise models exhibited similar residual and autocorrelation plots, whereas some third order noise models still had a significant autocorrelation coefficient at lag 1, opposite to the example in Figure 3c.

Figure 3. Residual analyses of the residuals from (a) exponential method C (representative for methods A and B) and Box-Jenkins transfer function (BJ-TF) models with noise model orders (b) [2 0] and (c) [3 2]. Left: residual values divided by the model parameter Amp, plotted in function of time. Right: autocorrelation coefficients up to lag 10.
**Table 2.** Model performance characteristics for identification of the noise model order. Dark squares show noise model orders that resulted in visually inappropriate full model fits. Upper values for every noise model structure are Young Identification Criterion (YIC) values, and lower values are normalised root-mean-squared error values (NRMSE) (%).

<table>
<thead>
<tr>
<th>Order $D(z^{-1})$</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Order $C(z^{-1})$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>-6.94 (2.31)</td>
<td>-5.69 (2.12)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.8 (1.3)</td>
<td>2.1 (1.1)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-6.82 (2.91)</td>
<td>-6.20 (2.11)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.5 (1.7)</td>
<td>2.0 (2.6)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-6.75 (2.63)</td>
<td>-6.21 (2.69)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2.1 (1.6)</td>
<td>2.0 (1.6)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Figure 4.** Example plot of VO$_2$ data and the BJ-TF system and full (system + noise) model. The estimated TD value was 20 s.

3.3. Kinetic Features

Figure 5 provides an overview of the VO$_2$ kinetic feature values as calculated with the different exponential (methods A, B and C) and full BJ-TF models. Methods A and B resulted in a smaller TD and a higher TC, MRT, Amp and SSG compared to method C and the BJ-TF full model (Figure 5). On a group level, method C and BJ-TF did not show significant differences between the kinetic feature values of the included patients (Figure 5). On an individual level, the median (± IQR) of the absolute differences between the feature values as calculated with method C and BJ-TF were 4 ± 13 s, 11 ± 17 s, 7 ± 8 s, 27 ± 43 ml·min$^{-1}$ and 0.5 ± 0.7 ml·min$^{-1}$·W$^{-1}$ for TD, TC, MRT, Amp and SSG, respectively. These absolute differences were respectively 24%, 20%, 9%, 4% and 4% of the respective median feature values as extracted from BJ-TF models. MRT values varied less than TC values between the different methods (Figure 5).

Two patients had extremely large kinetic feature values in all applied models (e.g., TC > 200 s and/or SSG > 20 ml·min$^{-1}$·W$^{-1}$). One of these patients cycled at a very low load during CWRT (22 W), the other patient showed a VO$_2$ response that increased almost linearly. When applying method A, one additional patient had extremely large kinetic feature values.
was mainly reflected in the TD values, which in turn greatly affected all other feature values (Figure 5).

The relatively small phase I contributions, compared to the other noise contributions, might be a result of the patients suffering from COPD [5] or because the test was preceded by unloaded cycling [19]. Averaging the VO2 response of several repetitions of the same time series, as could be seen in the VO2 time series, residual and autocorrelation plots (Figures 2 and 3). NRMSE values showed that the residuals acted on a magnitude of 10–12% of the total response amplitude, indicating that these noise components should not be neglected. Methods B and C had higher NRMSE values because they tried to surpass the contribution of the cardiodynamic phase (phase I), leading to higher residual values at the start of the test when the cardiodynamic phase plays an important role (Figures 2 and 3). However, method A is mechanistically unrealistic, as it does not assume any transit delays between muscle oxygen uptake and oxygen uptake at the mouth [19]. Only method C was able to fully discard the phase I contribution by removing the first 20 s of data. The ability to (partly) surpass the cardiodynamic phase was mainly reflected in the TD values, which in turn greatly affected all other feature values (Figure 5). This shows that for COPD patients, similar to healthy subjects [19,20], the applied exponential modelling method had a significant effect on the kinetic feature values.

Exponential models are most commonly used to examine VO2 kinetics [4]. However, none of the applied exponential modelling methods were able to account for the noise components in the VO2 time series, as could be seen in the VO2 time series, residual and autocorrelation plots (Figures 2 and 3). NRMSE values showed that the residuals acted on a magnitude of 10–12% of the total response amplitude, indicating that these noise components should not be neglected. Methods B and C had higher NRMSE values because they tried to surpass the contribution of the cardiodynamic phase (phase I), leading to higher residual values at the start of the test when the cardiodynamic phase plays an important role (Figures 2 and 3). However, method A is mechanistically unrealistic, as it does not assume any transit delays between muscle oxygen uptake and oxygen uptake at the mouth [19]. Only method C was able to fully discard the phase I contribution by removing the first 20 s of data. The ability to (partly) surpass the cardiodynamic phase was mainly reflected in the TD values, which in turn greatly affected all other feature values (Figure 5). This shows that for COPD patients, similar to healthy subjects [19,20], the applied exponential modelling method had a significant effect on the kinetic feature values.

Another strategy to exclude the phase I contribution might be applying a second-order exponential model, which aims to separately model the phase I and phase II contributions [4,20]. However, a second-order exponential model is based on the unproven assumption that the phase I response is exponential of nature [20]. Furthermore, the phase I contribution was not the only unwanted noise component in the data, as can be seen in Figure 3a. These other noise contributions acted at a similar level as phase I, making it difficult to distinguish phase I from other noise contributions (Figure 3a). The relatively small phase I contributions, compared to the other noise contributions, might be a result of the nature of the patients suffering from COPD.
of the patients suffering from COPD [5] or because the test was preceded by unloaded cycling [19].
Averaging the VO$_2$ response of several repetitions of the same exercise can cancel out the noise
contributions that are not related to the phase I contribution [21,22], but this requires unnecessarily
strenuous exercise sessions. Therefore, BJ-TF models were preferred because of their ability to account
for all unwanted components at once.

A [2 0] noise model structure was considered the most suitable for BJ-TF modelling. When considering
all second order noise models, NRMSE values were all at the same magnitude, and a [2 0] structure had
the best YIC values. This indicates that increasing the order of C(z$^{-1}$) above zero induced unnecessary
complexity. Some structure remained in the residuals when applying a second order noise model, but at
a very low magnitude (Figure 3b and Table 2). When increasing the noise model dynamics to order
three, this low-magnitude structure in the residuals disappeared in most patients (Figure 3c) and NRMSE
values became slightly lower, but were still at a similar magnitude as second order models (Table 2).
The improvements of increasing the noise model order were thus minimal compared to the cost of the
increased complexity (i.e., more difficult to interpret, greater risk of overfitting and more computationally
expensive), resulting in higher YIC values for third order noise models (Table 2). A [2 0] noise model showed
the best balance between model fit and model complexity.

BJ-TF models provided better fits than the exponential modelling methods. Whereas exponential
models only provided an estimate of the general trend of the VO$_2$ response (Figure 2), BJ-TF models
were able to account for the unwanted noise contributions around the general trend (Figure 5), without
having to average VO$_2$ responses over several repetitions [21,22]. These noise contributions are
especially important for COPD patients, as the reduced VO$_2$ responses, due to the lower workloads
that are feasible for COPD patients [3], increase the relative magnitude of the noise contributions
(i.e., lower signal to noise ratio). Residual and autocorrelation plots confirmed that the noise models
accounted for the most important noise contributions (Figure 3), and the remaining residuals were
four times smaller than residuals from exponential models (Figure 5). Whereas exponential modelling
models A and B failed to provide satisfying estimates of TD, BJ-TF models with a [2 0] noise model
always provided satisfactory results without having to manipulate the data manually (cf. method C).

The ability of BJ-TF models to separate the phase II contribution from other unwanted noise
contributions ensured that these noise contributions could not influence the extracted kinetic feature
values, which is a key advantage compared to exponential modelling. The influence of the unwanted
contributions, mostly phase I, led to lower TD and higher TC, MRT, Amp and SSG values when
applying methods A and B. Only method C produced similar feature values as BJ-TF models on a group
level, pointing out that this method should be preferred over methods A and B when exponential
models are applied on CWRT data from COPD patients. Nevertheless, median absolute differences
between the feature values of individual patients, as calculated with method C and BJ-TF models,
were up to 24%, showing that different feature values were obtained for individual patients when the
two different methods were applied. Unfortunately, due to the unwanted noise components in VO$_2$
responses, the true VO$_2$ kinetic feature values are never exactly known. As a result, no quantitative
evidence could be provided that indicates which model (method C or BJ-TF) was mainly responsible for
these differences in feature values on an individual level. Nevertheless, some theoretical considerations
favour BJ-TF models over exponential models, as discussed below.

Unwanted noise contributions influencing VO$_2$ kinetic features values are a general concern when
applying exponential models [22–24]. In contrast, BJ-TF models explicitly account for the unwanted
contributions and the extracted feature values will thus be less influenced by these contributions.
Furthermore, parameters in the exponential models were estimated using the generally applied method
of least squares, which can be influenced by unwanted noise contributions in the data, in contrast to the
more robust refined instrumental variable approach that was used to estimate the model parameters in
BJ-TF models [16,25]. In addition, BJ-TF models do not require data manipulation by removing the
first 20 s of data after the load onset. Although this method has been proven robust for changes in the
amount of removed data under specific conditions, e.g., using the averaged data of four repetitions of
the test and a sampling time of 10 s [26], the effect of changing the amount of removed data under less strict conditions, e.g., when only a single test is available, is still unclear and could thus influence the extracted feature values. Based on these elements, BJ-TF models are believed to be more reliable than exponential method C for the determination of kinetic feature values.

MRT varied less than TC over the different modelling methods for quantifying the speed of the primary VO$_2$ response (Figure 5). This confirms the findings from test–retest reliability studies that determine VO$_2$ kinetics from healthy subjects using a treadmill [27] or step test [28]. Inaccurate TD estimates will directly affect TC estimates in the opposite direction. These opposing effects are balanced in the calculation of MRT, making MRT more robust towards inaccurate TD estimations. This study also confirms earlier findings that not all CWRT datasets of COPD patients are well-suited for kinetic analyses [29]. One patient showed no meaningful VO$_2$ response due to a very low cycling load (22 W). More interestingly, another patient had an almost linear primary VO$_2$ response that was not due to a low cycling load. The airway and alveolar abnormalities in COPD patients, which can limit the oxygen delivery to the working muscles [4], might have led to this alteration of the VO$_2$ response. Analyses of larger COPD populations will be required to confirm this hypothesis.

In addition to the more reliable determination of kinetic feature values compared to exponential methods, A and B, BJ-TF models can also be applied more broadly than exponential models. Exponential models do not explicitly include an input variable in their model, limiting their use to experiments with a single input transition (e.g., non-active to active or unloaded to loaded cycling). BJ-TF, however, are able to estimate kinetic feature values with multiple input transitions, e.g., for time-domain analyses of pseudo-random binary sequences of the cycling load [30], or during unsupervised physical activities. As physical activity and VO$_2$ can nowadays be measured with wearable devices [31,32], combining these measurements with BJ-TF modelling can produce frequent estimates of kinetic feature values during standard unsupervised training sessions, without the need for elaborate testing procedures. Similar models have already been successfully used to model heart rate responses during road cycling [33].

Although this study showed the advantages of applying BJ-TF models, some considerations should be taken into account. We only examined a specific population, which exhibits a reduced signal to noise ratio (due to the reduced VO$_2$ response amplitudes [3]) and phase I contribution [5]. It would thus be interesting to test the performance of BJ-TF models in other population types (e.g., athletes or other disease states). Furthermore, other exercise modalities (e.g., treadmill walking) and exercise intensities should be examined to test the general applicability of BJ-TF models for analysing VO$_2$ kinetics.

In conclusion, low residual values showed that BJ-TF models accurately model the VO$_2$ response of COPD patients during a CWRT. These models, in contrast to the typically used exponential models, were able to account for unwanted noise contributions (e.g., the phase I contribution), leading to more reliable determinations of kinetic feature values compared to methods A and B. Only exponential modelling method C, which excludes the data during the cardiodynamic phase, led to comparable kinetic feature values as extracted from BJ-TF models. Based on theoretical considerations, we recommend using BJ-TF, rather than exponential models, for reliable determinations of VO$_2$ kinetics.


Funding: This research received no external funding.

Acknowledgments: This research is part of a PhD research funded by Flemish Institute for Technological Research (VITO), Mol, Belgium.

Conflicts of Interest: The authors declare no conflict of interest.
References


7. Palange, P.; Galassetti, P.; Mannix, E.T.; Farber, M.O.; Manfredi, F.; Serra, P.; Carlone, S. Oxygen effect on O\textsubscript{2} deficit and VO\textsubscript{2} kinetics during exercise in obstructive pulmonary disease. J. Appl. Physiol. 1995, 78, 2228–2234. [CrossRef] [PubMed]


17. Taylor, C.J.; Pedregal, D.J.; Young, P.C.; Tych, W. Environmental time series analysis and forecasting with the Captain toolbox. Environ. Model. Softw. 2007, 22, 797–814. [CrossRef]


